SINGLE ROW ROUTING WITH BOUNDED NUMBER OF DOGLEGS PER NET

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ABSTRACT

In this paper, we present sufficient and necessary conditions for single row routing when number of doglegs per net is bounded by $K, K = 1,2$ and develop algorithms for finding such realizations. The necessary and sufficient conditions are based on a graph theoretic representation, in which an instance of the single row routing problem is represented by three graphs, an overlap graph, a containment graph and an interval graph.

INTRODUCTION

The classical single row routing problem (SRRP) is an important problems in the layout design of multilayer circuit boards [10]. It has received considerable attention over the past ten years [1, 6, 9, 11]. The problem can be defined as follows: Given a set of two-terminal nets defined on a set of evenly spaced terminals on a line, called the node axis. The interconnections for the nets are to be realized by means of non-crossing paths. Each path consists of horizontal and vertical line segments on a single layer, such that no two paths cross each other. Moreover, no path is allowed to intersect a vertical line more than once, i.e. backward moves of nets are not allowed. The area above the node axis is called the upper street while the area below the node axis is called the lower street. The number of the horizontal tracks in the upper street is called the upper street congestion (C_{us}) and the number of horizontal tracks in the lower street is called the lower street congestion (C_{ls}). The objective function considered most often is to minimize the maximum of upper and lower street congestions, i.e. minimize Q_0, where $Q_0 = \max\{C_{us}, C_{ls}\}$.

Necessary and sufficient conditions for the optimal realization of single row routing problem are developed in [11]. Since the general problem is shown to be NP-Complete [9], several heuristic algorithms have also been proposed [2]. Finding a layout minimizing the total number of doglegs is one of the such problems that have significant applications [3, 9]. The related problem of finding the layout with restriction on the number of doglegs per net, is of particular interest to micro-wave circuit designers [8]. In [1] we presented necessary and sufficient conditions for existence of a routing without for a given SRRP. These conditions are based on a graph theoretic representation, in which an instance of the single row routing problem is represented by three graphs, a circle graph, a permutation graph and an interval graph [1].

In this paper we present sufficient and necessary conditions for existence of a realization for a single row routing problem when the number of doglegs per net is bounded by $K, K = 1,2$. Our basic discovery is that the necessity of doglegging in a realization for a net list is directly related to the structure of the containment graph G_C and the overlap graph G_O. The necessary and sufficient conditions for $K = 0$ is established by following theorem [1].

Theorem 1 *Given an instance of SRRP, a realization without doglegs exists if and only if its corresponding G_O is a bipartite graph.*

A GRAPH THEORETIC MODEL

In this section we briefly present a graph theoretic formulation of the single row routing problem. Details of this formulation are omitted from this summary and can be found in [1].

Define an *overlap graph* $\vec{G}_O = (V, \vec{E}_O)$,

$$V = \{v_i \mid v_i \text{ represents interval } I_i \text{ corresponding to } N_i\}$$

$$\vec{E}_O = \{(v_i, v_j) \mid l_i < l_j < r_i < r_j\}$$

Similarly, define a *containment graph* $\vec{G}_C = (V, \vec{E}_C)$, where the vertex set V is the same as defined above and $\vec{E}_C$ a set of directed edges defined below:

$$\vec{E}_C = \{(v_i, v_j) \mid l_i < l_j, r_i > r_j\}$$

We also define an interval graph $G_I = (V, E_I)$ where the vertex set V is the same as above, and two vertices are joined by an edge if and only if their corresponding intervals have a non-empty intersection. An example of the overlap graph, the containment graph and the interval graph for a set of nets can be found in [1].

If G_O contains more than one connected component then these components are related in a tree like fashion. To explore the relationship among the connected components of G_O we define a directed graph $\vec{T}_t$ as follows. Let $H = \{H_i \mid H_i = (V_i, E_i), 1 \leq i \leq r\}$ be the set of connected components of G_O. We define a directed graph $\vec{T} = (V^h, \vec{E}^h)$. where $V^h = \{v_1^h, v_2^h, \ldots, v_r^h\}$ such that there exist a bijection from H to V^h. The edge set $\vec{E}^h$ is defined as follows

$$\vec{E}^h = \{(v_i^h, v_j^h) \mid \exists u \in V_i, \exists v \in V_j, (u,v) \in \vec{E}_C, 1 \leq i,j \leq r\}$$

In other words, a directed edge is drawn from v_i^h to v_j^h if and only if the composite interval CI_{H_j} corresponding to H_j is completely contained in the composite interval CI_{H_i} correspond-

ing to H_i. Where composite interval of a connected component H_i is defined as $CI_{H_i} = (\min_k l_k, \max_k r_k)$ for all $k, N_k \in H_i$. The graph $\vec{T}_t = (V^h, \vec{E}_t^h)$ is then the transitively reduced graph corresponding to $\vec{T}$. In [1] we proved the following theorem.

Theorem 2 *$\vec{T}_t$ is a rooted tree.*

SUFFICIENT CONDITIONS FOR $K \leq 2$

Suppose there exist no realization of a net list with $K \leq 1$. This implies that in every realization of the net list at least one net is forced to have two doglegs. Sufficient conditions for such net lists can be developed by characterizing overlap graphs corresponding to the net list. A forbidden subgraph restriction on the overlap graph $\vec{G}_O$ leads us to following sufficient conditions.

Lemma 1 *At least one net must have two doglegs ($K = 2$) if $\vec{G}_O$ contains any one of the graphs in Fig. 1 as an induced subgraph.*

Proof: We show that every realization of a net list corresponding to the graph shown in Fig. 1(a) contains at least one net that is doglegged twice. The proof for other graphs shown in Fig. 1 is similar and is omitted.

First, we note that net list shown in Fig. 2(a) corresponding to graph 1(a) is minimum. This follows from the fact that there exists a routing with $K = 1$ if any one of the nets is deleted. Fig. 2(b) shows the routing if net 3a is missing.

We show the lemma by contradiction. We assume that for this net list $K = 1$ routing is possible and we attempt to construct the corresponding permutation. It is obvious that in any valid permutation net 1a must be placed outside net 1b in order to avoid double doglegging of net 1a. Similarly, net 2a must be placed outside net 2b. With relative positions of nets 1a, 1b, 2a and 2b fixed we can attempt to insert nets 3a and 3b. Again note that the only legal positions for 3a and 3b are between 1b and 2a. There are two cases: Suppose the permutation is (1a, 1b, 3a, 3b, 2b, 2a) In this case Net 3a is doglegged twice as shown in Fig 2(c). On the other hand, if the permutation (1a, 1b, 3b, 3a, 2b, 2a) is used then the net 3a violates the property $K = 1$. To summarize, in any routing of a three clique in the overlap graph at least one net is doglegged. The lemma follows from the observation that when two nets with containment relation are doglegged once, a double dogleg is forced in one of the two nets. □

NECESSARY CONDITIONS FOR $K \leq 1$

Our basic discovery is that the necessity of doglegging in a realization for a net list is related to the interaction between the containment and the overlap structures of the net list. Therefore we explore these two structures to investigate doglegging. In this section we develop a number of necessary conditions for $K = 1$ by restricting the structure of G_C and G_O .

Lemma 2 *If G_C is a null graph and G_O is a clique then there exists a realization with $K \leq 1$.*

Proof: We prove this lemma by construction. We sort all the nets in the net list according to their left end terminal. Let $S = s_1, s_2, \ldots, s_a$ be the sorted list, where $s_i, i = 1, 2, \ldots, a$ is some net in the net list.

Now suppose we have a tracks available for the layout. We assign each net s_i to the i^{th} track. In other words, the permutation specifying the layout is same as for the sorted list. An

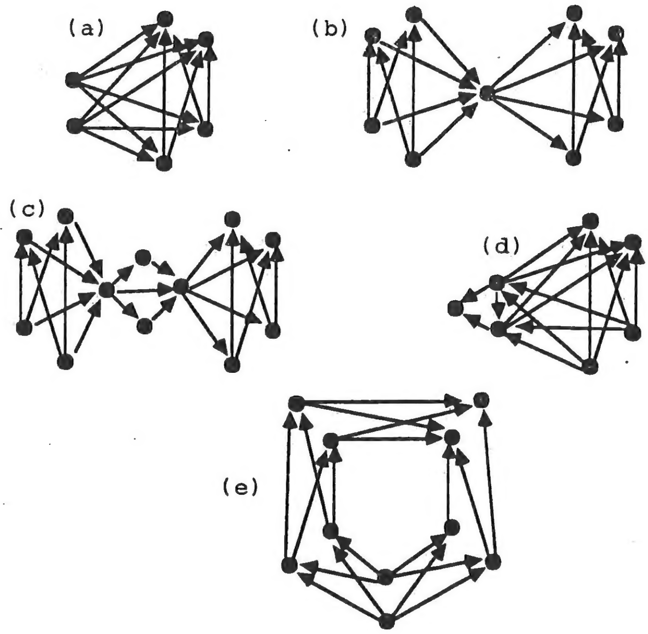


Figure 1: Forbidden subgraphs for Lemma 1

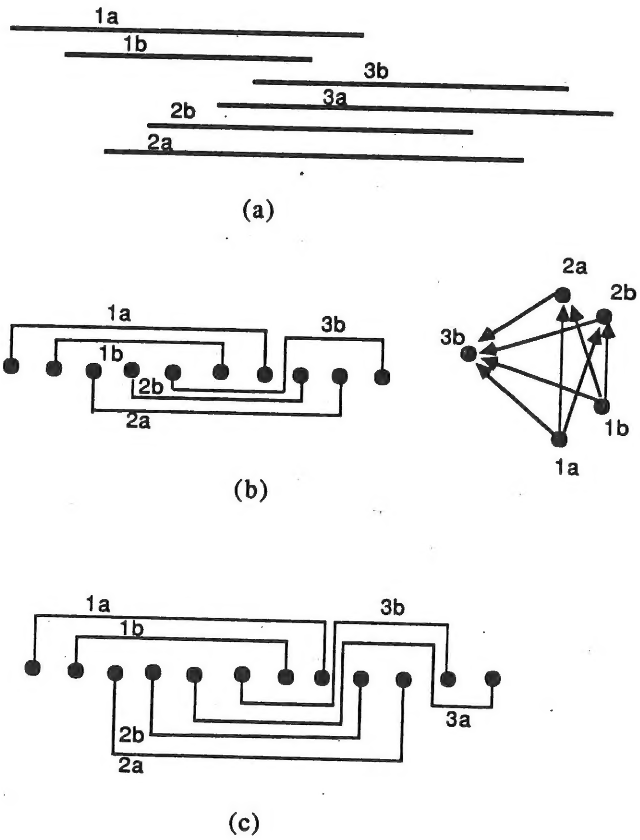


Figure 2: Figures for proof of Lemma 1

example of such track assignent is shown in Fig. 3.

It is clear that the method given above for routing generates a layout with $Q_0 = a - 1$. □

This method can be generalized to route a bigger class of SRRP as follows.

Theorem 3 *If G_C is a null graph then there exists a realization with $K \leq 1$.*

Proof: Let $C = C_1, C_2, \ldots, C_k$ be the linear ordering of the maximal cliques of G_I and let $S = s_1, s_2, \ldots, s_a$ be the list of nets in C_1, sorted according to their left end points. If G_O consists of only one clique then proof follows from theorem 2. Let T be a sorted list of nets in the clique C_2. Because G_C is null therefore the containment graph for the nets consisting of T must be null. This implies that the permutation T satisfies the property $K \leq 1$. Moreover, some nets in T also belong to C_1 therefore these nets are already present in the permutation S. Let $F = S \cap T = f_1, f_2, \ldots, f_c$ be the sorted list of these common nets. It is easy to see that $s_1 \notin T$. Let s_i be the first net that is also present in T. That is, s_i is a net in T such that i is minimum. It follows from the fact that the containment graph is null that for each net $f_j \in F, r_{s_{i-1}} < l_{f_j} < r_{s_i}$. Moreover, $r_{f_1} > r_{s_a}$, otherwise net f_1 would be contained in net s_a.

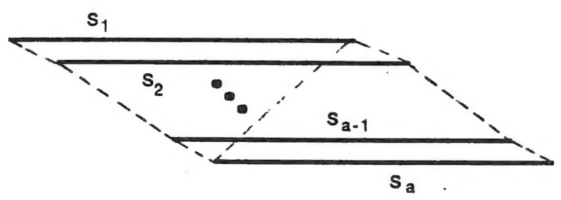
After routing of C_1, the reference line visits the terminals in the following order:

$$l_{s_1}, \ldots, l_{s_a}, r_{s_1}, \ldots, r_{s_{i-1}}, r_{s_i}, \ldots, r_{s_a}$$

It is east to see that nets belonging to F can be inserted into the permutation without violating the property of S that $K \leq 1$. We insert F into S just above s_i. After the insertion the reference line visits the terminals in the following order.

$$l_{s_1}, \ldots l_{s_a}, r_{s_1}, \ldots r_{s_{i-1}}, l_{f_1}, \ldots, l_{f_c}, r_{s_i}, \ldots, r_{s_a}, r_{f_1}, \ldots, r_{f_c}$$

(a)



(b)

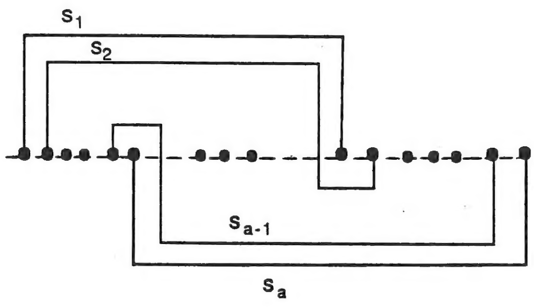


Figure 3: Routing with at most one dogleg

This insertion method leads to the routing as shown in Fig. 4. It is clear from Fig. 4. that insertion of new nets does not add any doglegs to previously routed nets. In addition, the nets in F do not violate the $K \leq 1$ property. Thus the new routing also satisfies the $K \leq 1$ property. Other cliques, $C_3, C_4, \ldots, C_k$ can be inserted in S in a similar manner. □

We now present an algorithm ONEDOGLEGI which routes a given SRRP with at most one dogleg per net if the corresponding containment graph to the given SRRP is null.

Algorithm ONEDOGLEGI( )

Input: A Net list L with null G_C , and a list $C = C_1, C_2, \ldots C_k$, giving a linear ordering of the maximal cliques of G_I .
Output: Permutation S of the given Net list with at most one dogleg per net

1. Sort the nets in the clique C_1 according to their l_i's. Let $S = s_1, s_2, \ldots, s_a$ be the sorted list.

2. Repeat

   2.1 Let $N' \neq s_1$ be the net with minimum $r_{N'}$ among the nets that have been already assigned a position in the permutation but whose right end has not been considered. If such a net is not found then set $N' = 0$

   2.2 Let NL be the set of nets N_i such that $l_i < r_{N'}$ and N_i is not yet assigned a position in the permutation.

   2.3 Sort nets in NL according to their l_i's Let $T = t_1, t_2, \ldots, t_b$ be the sorted list

   2.4 Insert T into S one position above N'. Let S be the new permutation after the insertion.

3. Until $N' = 0$

Algorithm ONEDOGLEGI can be improved so that it can find a solution of a given SRRP if G_O has more than one connected components, and the containment graph of each connected component is null. We present an algorithm ONEDOGLEGII which first finds all the connected components.

Algorithm ONEDOGLEGII( )

Input: A Net list with null G_C .
Output: Permutation of the given Net list with at most one dogleg per net

1. Form G_O . If given SRRP has only one connected component then call ONEDOGLEGI(H_1, P_1) . Stop. Else form the parent tree $\vec{T_t}$.

2. For each connected component H_i do call ONEDOGLEGI(H_i, P_i)

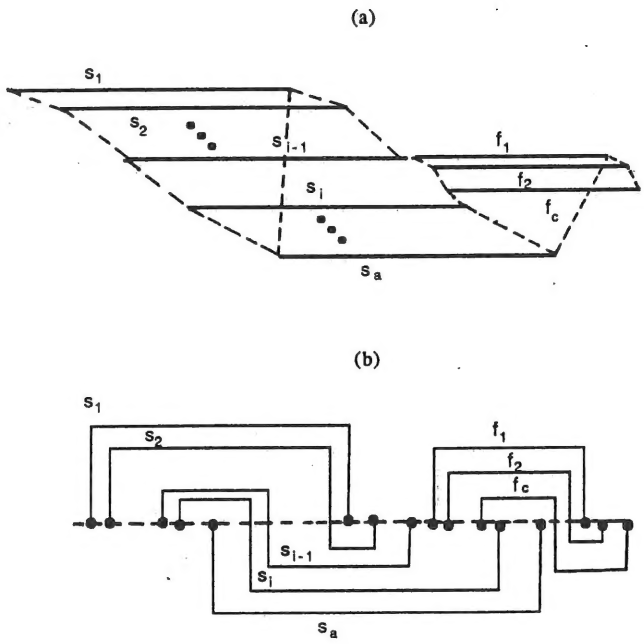


Figure 4: Figure for proof of Theorem 3

3. While not empty($\vec{T_t}$) do
    - 3.1 Find a node H_i whose children have zero out-degree.
    - 3.2 For each Child H_{ij} of H_i do
        - 3.2.1 Find a valid insertion position t in P_i
        - 3.2.2 Insert $P_{H_{ij}}$ into P_i
    - 3.3 Delete each Child H_{ij} of H_i
    - 3.4 If H_i is root then delete H_i

The following theorem establishs the correctness of the algorithm ONEDOGLEGII.

Theorem 4 *Let $G^i_C, 1 \leq i \leq r$ be the containment graph induced by the vertex set of each of the connected components of G_O . If G^i_C is a null graph for each i then algorithm ONEDOGLEGII finds a realization of the given net list with $K \leq 1$.*

An example of routing by Algorithm ONEDOGLEGII is shown in Fig. 5.

The sufficient condition in Lemma 1 and minimality of graph 1(a) leads to necessary conditions in the following lemmas. The proof is similar to the proof of Lemma 1.

Lemma 3 *If at least one net is forced to have two doglegs in every realization of a net list then $|E_C| \geq 3$.*

Lemma 4 *If at least one net is forced to have two doglegs in every realization of a net list then G_O contains $K_{2,2}$ as an induced subgraph.*

Lemma 5 *If a net is forced to have two doglegs ($K = 2$) in every realization of a net list then $|C_I| \geq 5$.*

Lemma 6 *If at least one net is forced to have two doglegs ($K = 2$) in every realization of a net list then the net list has at least five nets.*

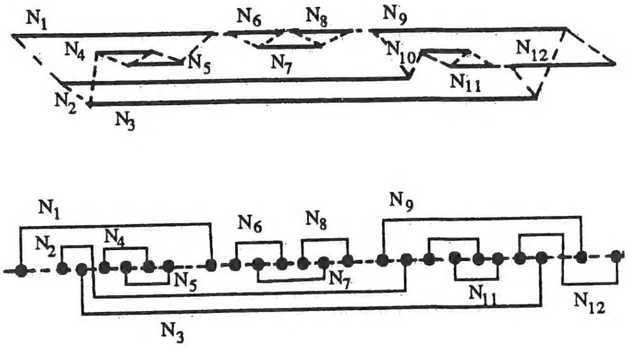


Figure 5: Realization with $K = 1$ when G^i_C is null.

CONCLUSIONS

In this paper, we have presented sufficient and necessary conditions for single row routing when number of doglegs per net is bounded by $K, K = 1, 2$ and develop algorithms for finding such realizations. The necessary and sufficient conditions are based on a graph theoretic representation developed by authors in earlier paper [1].

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